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EUROPEAN LONGITUDE NETWORK AND A PROJECT FOR BELGRADE INCLUSION

Z. CVETKOVIĆ
Astronomical Observatory, Volgina 7, 11160 Belgrade 74, Serbia
E–mail zcvetkovic@aob.bg.ac.yu

Abstract. The first part of this work is devoted to the investigation of mathematical models for adjusting observations, describing influences of various factors on measurements during the longitude and the longitude difference determinations. Among 12 investigated models, it the model showing the statistically best agreement with the measurement data is accepted. The second part is devoted to the examination of the influence of stellar positions on such determinations during the use of two celestial reference frames: the dynamical one, determined by the FK5 catalogue and the kinematical one, determined by the Hipparcos catalogue. It has been determined the systematic difference of two mentioned bases for the longitude determination, which is annulated in the case of longitude differences. In the third part, the functional model, providing the satisfying precision and high reliability for the Belgrade Astronomical Observatory inclusion in the European longitude network, has been investigated. Such point could be a reference point for the determination of the national longitude network and for the geoid determination for our country. By changing the geometry of a part of ELN, it has been found that this could be achieved by the determination of the Belgrade longitude difference related to only two nearly stations, members of ELN.

1. INTRODUCTION

The determination of the astronomical geoequatorial longitude is based, on the one hand, on the observations of stars with respect to the local plumb line and, on the other hand, on the positions of these stars in a fundamental catalogue which materialise the celestial reference system. Therefore, for the purpose of estimating the accuracy and reliability of longitude determination it is necessary to estimate influences due to the changes of the celestial reference system and to the random and systematic errors in the positions of observed stars in view of the determination methods.

Activities on the formation of the European Longitude Network (ELN) took place in the late XX century (Kaniuth et al. 1988). It was developed for the purpose of an accurate and homogeneous astronomical reference system necessary in both a common adjustment of the European Trigonometric Network and the astrogeodetic and astrogravimetric geoid determination.

Each of the member countries in the European Trigonometric Network should have at least one station in ELN. The campaign for including Astronomical Observatory in

Belgrade (AOB) in ELN, planned for 1990, has not been realised even till nowadays.

For the purpose of ELN establishing in the period 1977-1980, i.e. 1988 precise measurements of longitude differences between national reference stations in Germany, Italy, Spain, the Netherlands, France, Portugal and Austria (Kaniuth and Wende 1980; 1983; Wende 1992) took place. These measurements were done with a Danjon astrolabe by applying the method of equal zenith distances. The whole observational material was put at the present author's disposal due to the courtesy of Academician Prof. Dr R. Sigl and Dr W. Wende.

The last campaign for the longitude-difference determining in the framework of ELN was carried out in 1988. The longitude differences were determined between two Austrian stations, Vienna and Graz and the reference station in Munich. The observer was W. Wende. He observed selected stars from the FK5 Catalogue with declinations between 20^o and 70^o. The measurements from the 1988 campaign are treated and analysed in the present paper.

2. MATHEMATICAL ADJUSTMENT MODEL

The mathematical model for the adjustment of observations requires the relative weights of observations to be determined previously. The study of a weight model is equivalent to the study of a model of variance-components measurements.

Here four models of variance components (VC) are used (Perović and Cvetković 2001):

1. zenith-distance variance as a function of time-registration variance - model VC1; 2. Wende's model of variance components - model VCW; 3. two-component model - VC2 model and 4. three-component model - model VC3.

Due to a common adjustment of all campaign measurements an adequate functional regression model is studied. The functional model of the covariance analysis, i.e. correction equations, is used:

$$\mathbf{v} = \mathbf{A}\mathbf{x} + \mathbf{C}\mathbf{z} + \mathbf{f} \tag{1}$$

where $\mathbf{v}$ is the vector of observation corrections, $\mathbf{A}$ and $\mathbf{C}$ are the matrices of known coefficients of the linear model, $\mathbf{x}$ is the vector of unknown basic parameters, $\mathbf{z}$ is the vector of unknown additional parameters and $\mathbf{f}$ is the vector of free terms of the linear model.

The vector of basic (obligatory) parameters $\mathbf{x}$ is the same in each adjusting model and it has nine components: three increments $d\varphi_j$, $(j = 1, 2, 3)$, three increments $d\lambda_j$, $(j = 1, 2, 3)$ where $j = 1$ corresponds to Station Munich, $j = 2$ to Station Vienna, $j = 3$ to Station Graz and three increments dz_k, $(k = 1, 2, 3)$ for three groups of observed stars 10, 11 and 12 ($k = 1$ for Group 10, $k = 2$ for Group 11, $k = 3$ for Group 12). For this reason the functional models under study differ in the term $\mathbf{Cz}$ representing influences of various factors on the observations.

Besides, with each functional model four adjustment versions are obtained depending on which model of observation weights is used.

2.1. THE FIRST FUNCTIONAL MODEL - FM1

This model is used for describing the influences of eight factors on the observations so that the FM1 model has a total of 17 parameters, i.e. every observation has the following correction equation:

$$
\begin{aligned}
l_p \;+\; v \;=\; & b_\varphi d\varphi_j \;+\; b_\lambda d\lambda_j \;-\; dz_k \\
& + \Delta T b_\varphi \Delta\varphi_G \;+\; \Delta T b_\lambda \Delta\lambda_G \\
& + b_{IT} dIT \;+\; b_{IA} dIA \;+\; b_{HD_2} dHD_2 \\
& + b_H dH1 \;+\; b_H^2 dH2 \;+\; b_F dF
\end{aligned}
\tag{2}
$$

where the nine obligatory parameters are in the first row. The eight additional parameters are in the other rows: $\Delta\varphi_G$ - time variation of latitude, $\Delta\lambda_G$ - time variation of longitude, dIT - correction for the variation of the instrument temperature, dIA - correction for the variation of the temperature difference between the instrument and the environment, dHD_2 - correction for the human-eye adaptation to light and darkness, $dH1$ and $dH2$ - corrections for the influence of the star apparent magnitude and dF - correction for the influence of star colour.

After adjusting by using the FM1 model the estimates of the corrections $\hat{v}$ are analysed with respect to various regressors (influences): series, nights, azimuth, apparent magnitudes of stars, outer air temperature measured during the observations, atmospheric pressure and instrument temperature. The distribution of correction estimates $\hat{v}$ is given in Figs. 1-7 (left). In these figures the individual correction estimates are presented as dots, whereas the mean values are presented with circles connected in a line. It is seen that there is a systematic variation from series to series, i.e. from night to night, caused, most likely, the varying of the outer temperature and of that of the instrument with night. On account of this instead of two parameters which describe these influences (dIT and dIA) over the entire campaign 23 different ones are introduced, one for each night, i.e. dIT_h and dIA_h, $h = 1, ..., 23$ (Cvetković and Perović 2000) and in this way the functional model FM2 is formed.

2.2. THE SECOND FUNCTIONAL MODEL - FM2

In this adjusting model, with a total of 61 parameters, each observation has the following correction equation:

$$
\begin{aligned}
l_p \;+\; v \;=\; & b_\varphi d\varphi_j \;+\; b_\lambda d\lambda_j \;-\; dz_k \\
& + \Delta T b_\varphi \Delta\varphi_G \;+\; \Delta T b_\lambda \Delta\lambda_G \\
& + b_{IT} dIT_h \;+\; b_{IA} dIA_h \;+\; b_{HD_2} dHD_2 \\
& + b_H dH1 \;+\; b_H^2 dH2 \;+\; b_F dF
\end{aligned}
\tag{3}
$$

where the nine obligatory parameters are in the first row as in the case of FM1; the other rows contain 52 additional parameters out of which there are six in common

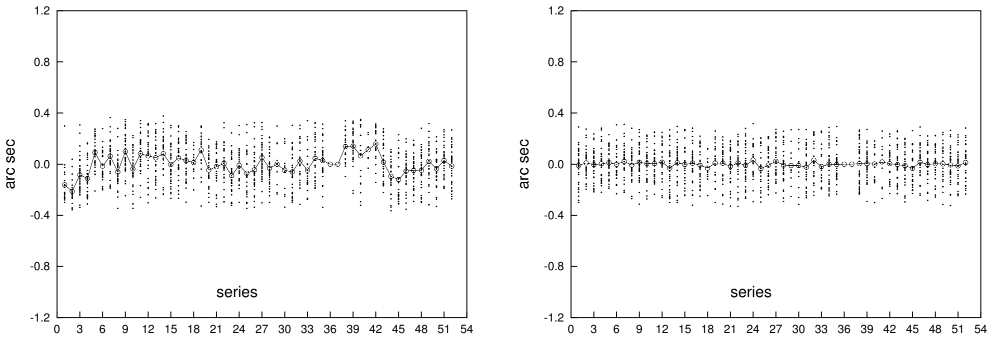

Figure 1: The distribution of correction estimates $\hat{v}$ in series obtained by using FM1 model (on the left) and FM2 (right).

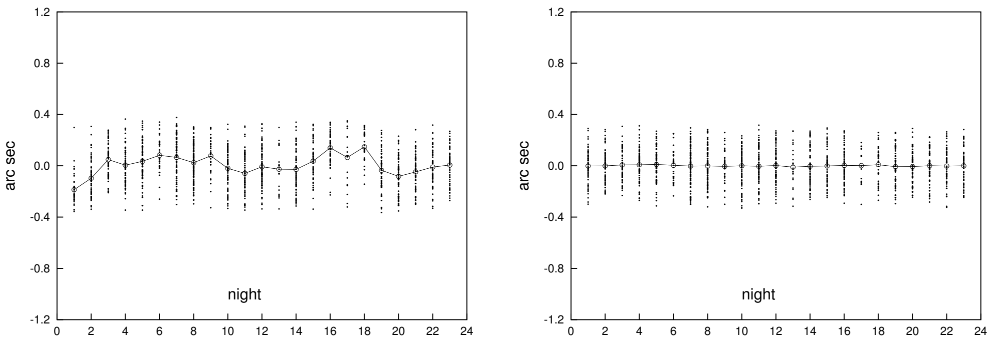

Figure 2: The distribution of correction estimates $\hat{v}$ in nights obtained by using FM1 model (on the left) and FM2 (right).

with FM1. The other 46 additional parameters ($dIT_h,\ dIA_h,\ \ h = 1, ..., 23$) (two for each of 23 observation nights) are introduced instead of two ones (dIT and dIA) for the entire campaign.

After adjusting with the FM2 model the correction estimates $\hat{v}$ are analysed with respect to the same regressors (influences) as the correction estimates obtained after the adjusting by using the FM1 model. The distribution of the correction estimates $\hat{v}$ is given in Figs. 1-7 (right). In these figures the systematic change from night to night cannot be longer seen. This indicates that the augmentation of the model with eight additional parameters to 52 is justified, i.e. the FM2 model yields a better description of the influences of some factors on the observations. The values obtained for $\hat{\mathbf{v}}^T\mathbf{P}\hat{\mathbf{v}}$ (Table 1) are also in favour of this justification. They are by about 35% smaller compared to the corresponding values for the model FM1. Something similar can be also said for the estimates of the variances m_o^2.

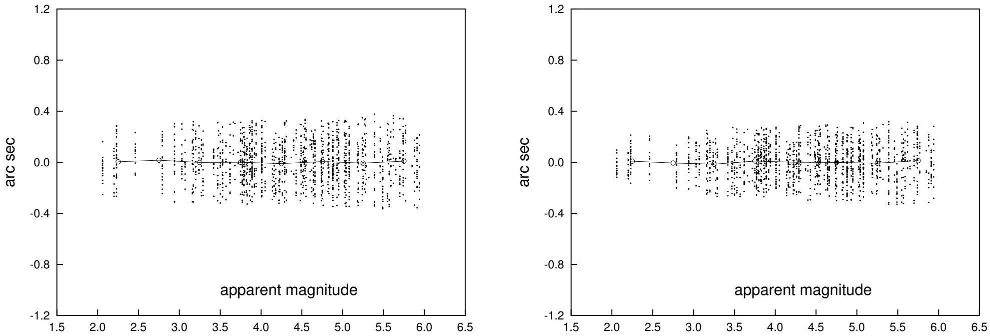

Figure 3: The distribution of correction estimates $\hat{v}$ in apparent magnitude obtained by using FM1 model (on the left) and FM2 (right).

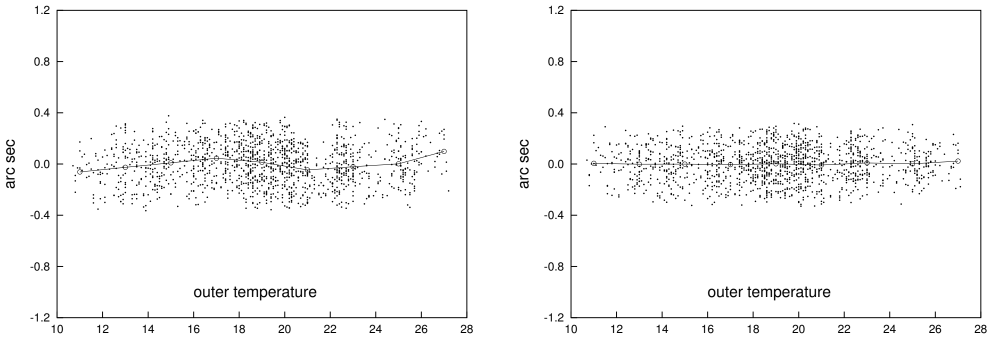

Figure 4: The distribution of correction estimates $\hat{v}$ in outer temperature obtained by using FM1 model (on the left) and FM2 (right).

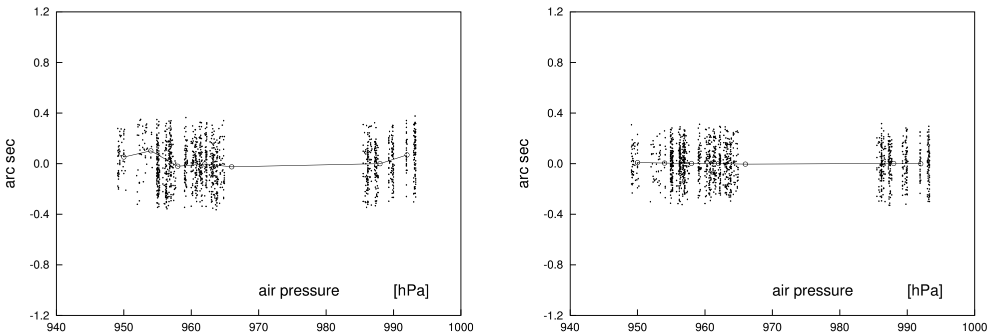

Figure 5: The distribution of correction estimates $\hat{v}$ in air pressure obtained by using FM1 model (on the left) and FM2 (right).

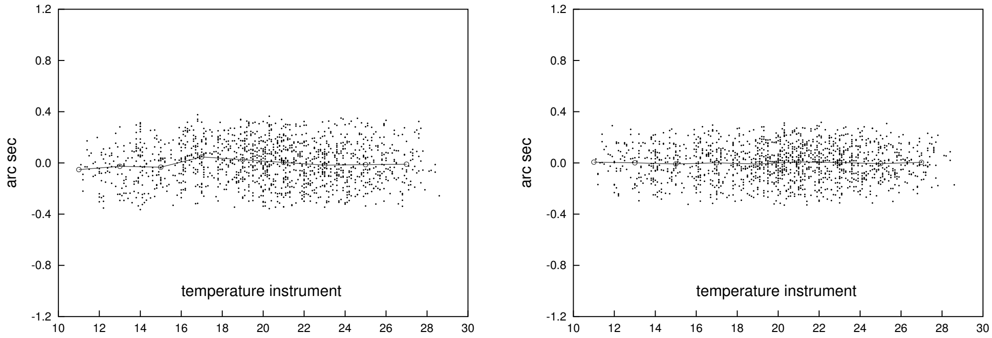

Figure 6: The distribution of correction estimates $\hat{v}$ in temperature instrument obtained by using FM1 model (on the left) and FM2 (right).

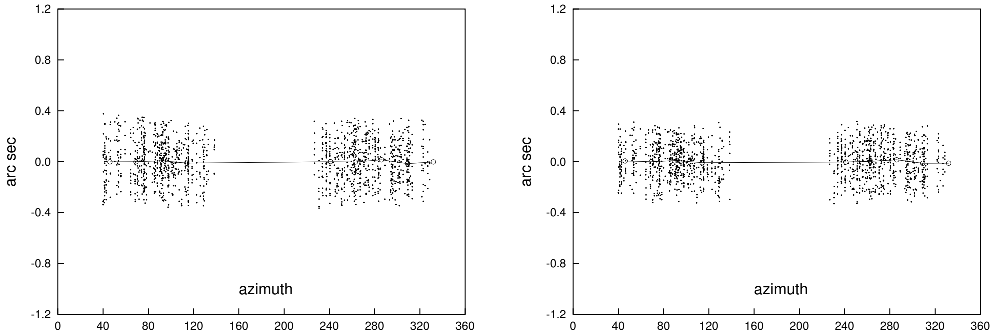

Figure 7: The distribution of correction estimates $\hat{v}$ in azimuth obtained by using FM1 model (on the left) and FM2 (right).

2.3. THE THIRD FUNCTIONAL MODEL - FM3

Compared to the FM2 model the vector of additional parameters **z** is augmented to comprise more parameters which represent corrections of the right ascensions of stars $d\alpha_\nu$, $\nu = 1, ..., m$ where the subscript ν is the number of stars for which right-ascension corrections are necessary.

In the FM3 model the equations of observation corrections have the form:

$$\begin{aligned} l_p + v = \; & b_\varphi d\varphi_j + b_\lambda d\lambda_j - dz_k \\ & + \Delta T b_\varphi \Delta\varphi_G + \Delta T b_\lambda \Delta\lambda_G \\ & + b_{IT} dIT_h + b_{IA} dIA_h + b_{HD_2} dHD_2 \\ & + b_H dH1 + b_H^2 dH2 + b_F dF \\ & + \sum_{\nu=1}^{m} (-b_\lambda \; d\alpha_\nu) \end{aligned} \tag{4}$$

Table 1: The adjustment results obtained by using three functional models: FM1, FM2 and FM3: P_{KD1} - weights calculated from model KD1; P_{KDW} - weights calculated from model KDW; P_{KD2} - weights calculated from model KD2; P_{KD3} - weights calculated from model KD3; n - number of observations; u - number of model parameters; $\hat{\mathbf{v}}^T\mathbf{P}\hat{\mathbf{v}}$ - sum of correction squares, m_o^2 - variance estimate and f - number of freedom degrees.

Func. Model	Weights	n	u	$\hat{\mathbf{v}}^T\mathbf{P}\hat{\mathbf{v}}$ $[''^2]$	m_o^2 $[''^2]$	f
$FM1$	P_{KD1}	1375	17	38.4551	0.02832	1358
	P_{KDW}	1378	17	37.7076	0.02771	1361
	P_{KD2}	1378	17	36.1434	0.02656	1361
	P_{KD3}	1378	17	34.9085	0.02565	1361
$FM2$	P_{KD1}	1377	61	25.8567	0.01965	1316
	P_{KDW}	1377	61	24.5164	0.01863	1316
	P_{KD2}	1376	61	23.8305	0.01812	1315
	P_{KD3}	1396	61	21.0608	0.01578	1335
$FM3$	P_{KD1}	1375	97	21.6668	0.01695	1278
	P_{KDW}	1377	94	19.5538	0.01524	1283
	P_{KD2}	1377	97	18.2303	0.01424	1280
	P_{KD3}	1377	95	17.4259	0.01359	1282

where the a priori unknown (variable) number m of right-ascension corrections $d\alpha_\nu$ $(\nu = 1, ..., m)$ is in the last row.

At first for every star the right-ascension correction $d\alpha$ is introduced to apply the method of gradual exclusion afterwards. If at the significance level of 0.05, the test statistic does not indicate existence of $d\alpha$ for some star, this parameter is omitted in the next iteration, i.e. the number of additional model parameters is diminished by one.

The final results of adjustment for all the three functional models and their mutual comparison are given in Table 1.

The models of variance components KD1 and KDW, with regard that they have one variance component, the former one for observation, the latter one for a star group, belong to the so-called models yielding a priori positive estimates of variance components. The models KD2 and KD3 yield positive estimates for the variance components only in the case of using the functional model FM3 supposed to describe well the influences of the factors active in the observations, i.e. it is thought to be adequate. The fact that the values obtained for $\hat{\mathbf{v}}^T\mathbf{P}\hat{\mathbf{v}}$ and the estimates of the variances m_o^2 for this functional model are by about 20% smaller compared to the functional model FM2, where the variance estimates are by about 35% smaller than in the case of the functional model FM1, is in favour of this.

Table 2: Longitude values λ for three stations; σ_λ are the errors of longitude determinations.

Station	λ [h m s]	σ_λ [s]
Munich	0 46 16.8747	0.00064
Vienna	1 05 20.9012	0.00076
Graz	1 01 58.5779	0.00080

Table 3: Values of longitude differences $\Delta\lambda$; $\sigma_{\Delta\lambda}$ are the errors of the longitude-difference determination.

From – To	$\Delta\lambda$ [h m s]	$\sigma_{\Delta\lambda}$ [s]
Munich – Graz	−0 15 41.7032	0.00090
Graz – Vienna	−0 03 22.3233	0.00104
Vienna – Munich	0 19 04.0265	0.00096

It is concluded that the use of the FM3 model yields the best description of the influences of the factors present in the observations, i.e. it is justified to introduce the right-ascension corrections.

In the case of all the three functional adjusting models the smallest variance estimates m_o^2 are obtained when the P_{KD3} model for observation weights is applied. These results indicate that it is justified to introduce the third variance component describing the influence of the star apparent magnitude on the observation accuracy. Among the four examined models of variance components, i.e. weights of observations, KD3 yields the best results, as seen from Table 1, so it is thought to be adequate.

On the basis of this analysis the best mathematical model is chosen. This is the functional model FM3 with the weight model P_{KD3}.

With this mathematical model and by using the star positions from the HIPPARCOS Catalogue the longitudes (Table 2), as well as the longitude differences (Table 3), for the three stations participating in the campaign - Munich, Vienna and Graz - are determined.

3. COMPARISON OF ADJUSTMENT RESULTS OBTAINED BY USING STAR POSITIONS FROM TWO CATALOGUES, FK5 AND HIPPARCOS

The influence of star positions on the determination of longitudes and their differences is examined. For this purpose the measurements of the whole campaign are adjusted also by applying the positions of the observed stars in the fundamental catalogue FK5. The functional model FM3 is used, whereas the weights are from the model of variance components KD3.

The results of the longitude determination are given in Table 4, the corresponding

Table 4: Values of longitudes λ for three stations: σ_λ are errors of longitude determination; $\lambda_{FK5} - \lambda_{Hipp}$ are longitude differences obtained by using star positions from catalogues FK5 and HIPPARCOS.

Stations	λ [h m s]	σ_λ [s]	$\lambda_{FK5} - \lambda_{HIPP}$ [s]	$\sigma_{\lambda_{FK5}-\lambda_{HIPP}}$ [s]
Munich	0 46 16.8778	0.00065	0.0031	0.00091
Vienna	1 05 20.9041	0.00077	0.0029	0.00108
Graz	1 01 58.5809	0.00079	0.0030	0.00112

Table 5: Values of longitude differences $\Delta\lambda$; $\sigma_{\Delta\lambda}$ are determination errors for longitude differences; $\Delta\lambda_{FK5} - \Delta\lambda_{HIPP}$ are longitude differences obtained by using star positions from catalogues FK5 and HIPPARCOS.

From – To	$\Delta\lambda$ [h m s]	$\sigma_{\Delta\lambda}$ [s]	$\Delta\lambda_{FK5} - \Delta\lambda_{HIPP}$ [s]
Munich – Graz	−0 15 41.7031	0.00091	0.0001
Graz – Vienna	−0 03 22.3232	0.00104	0.0001
Vienna – Munich	0 19 04.0263	0.00096	0.0002

results concerning the longitude differences are given in Table 5. In these tables the differences following from the values obtained with the HIPPARCOS positions are also given.

It is seen from Table 4 that the longitude differences $\lambda_{FK5} - \lambda_{HIPP}$ are equal for all the three stations, i.e. their amount is 0.0030 seconds of time which is due to the systematic difference between the two reference frames.

The catalogues FK5 and HIPPARCOS have a small rigid-body residual rotation which can be examined from a comparison of the positions and proper motions of the fundamental stars in the two catalogues. The vector of the orientation difference between the two reference frames can be determined from the difference of the positions, whereas the difference of the proper motions offers the possibility to determine the vector of the rotation difference between the two reference frames. The preliminary results obtained from the catalogue differences for all 1535 stars of the fundamental FK5 Catalogue can be found in the Foreword to the HIPPARCOS Catalogue and they are referred to the epoch J1991.25.

The rigid-body rotation (no coinciding of celestial coordinate directions), i.e. the differences in the star positions between FK5 and HIPPARCOS, affects the longitude determination, but not that of their differences which is seen from Table 5. The differences $\Delta\lambda_{FK5} - \Delta\lambda_{HIPP}$ are practically zero.

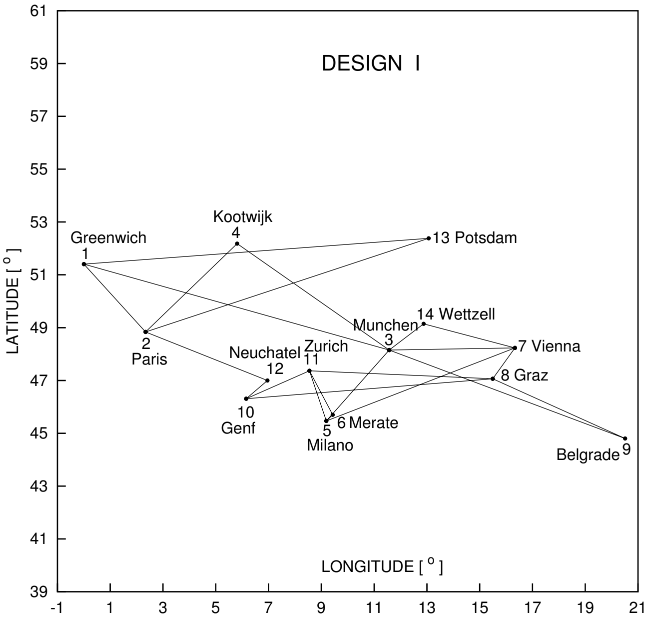

Figure 8: Design I.

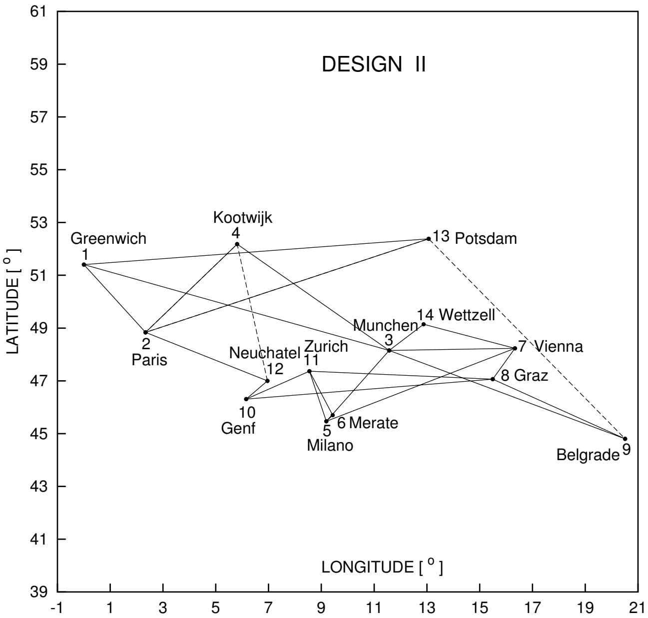

Figure 9: Design II.

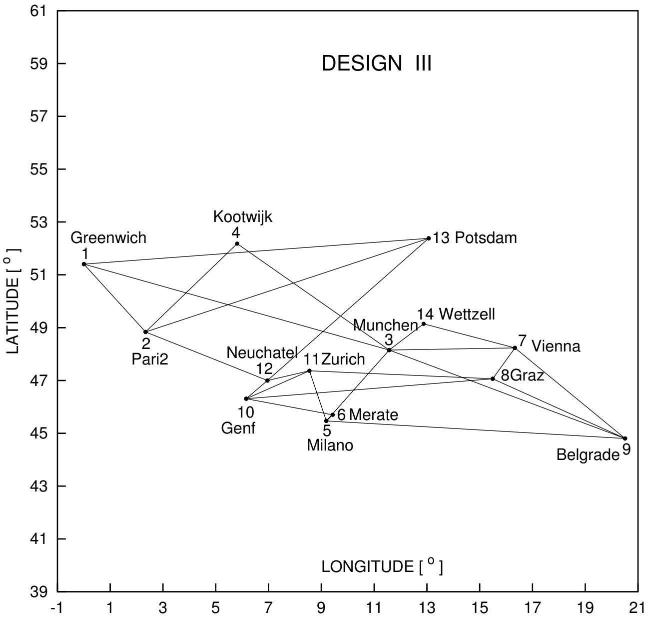

Figure 10: Design III.

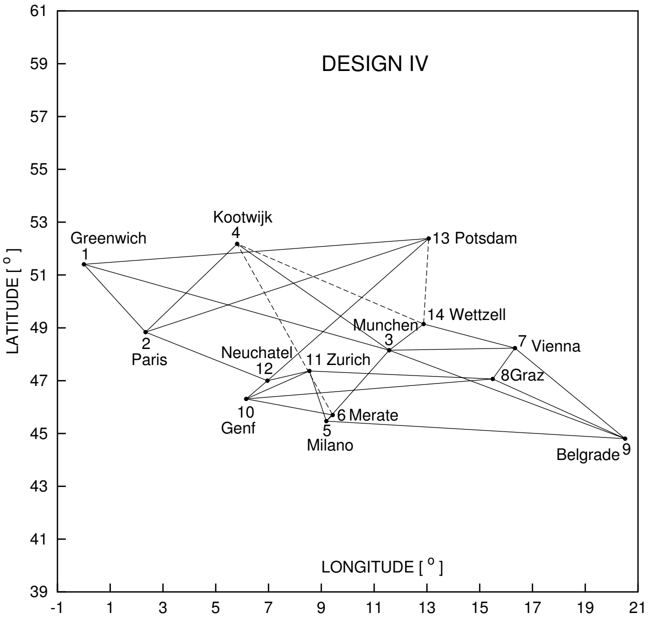

Figure 11: Design IV.

Table 6: Values of reliability measures r_{ii}, G_i^*, $\frac{G_i^*}{\sigma_i}$ and $\sqrt{\lambda_{x,i}}$ longitude differences for Design I.

i	From	To	r_{ii}	G_i^*	G_i^*/σ_i	$\sqrt{\lambda_{x,i}}$
1	1	13	0.2666	0.06713	4.615	4.154
2	13	2	0.2666	0.06713	4.615	4.154
3	2	4	0.3164	0.06163	4.236	3.564
4	4	3	0.3164	0.06163	4.236	3.564
5	3	7	0.6993	0.05116	3.517	2.264
6	7	8	0.5221	0.05676	3.901	3.001
7	8	9	0.3240	0.06090	4.186	3.483
8	9	3	0.3240	0.06090	4.186	3.483
9	3	5	0.5057	0.06056	4.162	3.445
10	5	6	0.3665	0.06798	4.673	4.241
11	6	11	0.4085	0.06798	4.673	4.241
12	11	10	0.5761	0.05932	4.077	3.303
13	10	12	0.2289	0.07244	4.980	4.692
14	12	2	0.2289	0.07244	4.980	4.692
15	2	1	0.6301	0.05258	3.614	2.464
16	1	3	0.4178	0.06021	4.139	3.406
17	3	14	0.3234	0.06095	4.190	3.489
18	14	7	0.3234	0.06095	4.190	3.489
19	7	5	0.4476	0.06245	4.293	3.656
20	5	11	0.4583	0.06193	4.257	3.598
21	11	8	0.5416	0.05673	3.900	2.998
22	8	10	0.5078	0.05802	3.988	3.152

It should be said that in this campaign only one segment of the reference frame is used. The observing programme contains 121 FK5 stars only (out of total of 1535) being less than 10%. Besides, these stars are from one part of the celestial sphere. Their declinations are between 20^o and 70^o, whereas the right ascensions are between 14 and 20.5 hours, i.e. 20 and 25.5 hours for western and eastern transits, respectively. Despite all of this a constant longitude difference $\lambda_{FK5} - \lambda_{HIPP}$ for all the three stations is obtained. This is, on the one hand, a confirmation of the rigid-body rotation between the two reference frames and, on the other hand, a confirmation that the used functional adjustment model and that of observation-weight determination are adequate.

4. THE PROPOSAL OF A PROJECT FOR INCLUDING BELGRADE IN THE ELN NETWORK

One of the objectives, which should have been realised as early as about fifeteen years ago, is to include one of our national stations in the ELN network. The most suitable station for this purpose is the Astronomical Observatory in Belgrade (AOB)

Table 7: Values of reliability measures r_{ii}, G_i^*, $\frac{G_i^*}{\sigma_i}$ and $\sqrt{\lambda_{x,i}}$ longitude differences for Design II.

i	From	To	r_{ii}	G_i^*	G_i^*/σ_i	$\sqrt{\lambda_{x,i}}$
1	1	13	0.3562	0.06439	4.426	3.866
2	13	2	0.4498	0.05986	4.115	3.366
3	2	4	0.5309	0.05447	3.744	2.715
4	4	3	0.4328	0.05799	3.986	3.149
5	3	7	0.7014	0.05100	3.506	2.241
6	7	8	0.5280	0.05641	3.878	2.958
7	8	9	0.4184	0.05862	4.029	3.222
8	9	3	0.5033	0.05544	3.811	2.838
9	3	5	0.5091	0.06028	4.144	3.414
10	5	6	0.3682	0.06770	4.654	4.213
11	6	11	0.4091	0.06770	4.654	4.213
12	11	10	0.5852	0.05878	4.041	3.242
13	10	12	0.3405	0.06734	4.629	4.176
14	12	2	0.4385	0.06200	4.262	3.605
15	2	1	0.6251	0.05238	3.600	2.436
16	1	3	0.4930	0.05670	3.897	2.993
17	3	14	0.3240	0.06090	4.186	3.483
18	14	7	0.3240	0.06090	4.186	3.483
19	7	5	0.4485	0.06233	4.285	3.643
20	5	11	0.4604	0.06176	4.245	3.579
21	11	8	0.5424	0.05670	3.897	2.994
22	8	10	0.5337	0.05702	3.919	3.033
23	12	4	0.3587	0.07939	5.457	5.366
24	13	9	0.3187	0.08377	5.758	5.778

because it was a reference station to astrogeodetic determinations in the framework of the activities aimed at the formation of an astrogeodetic network for our country and, besides, it was a member of BIH during a long time.

This would be of importance to the formation of the basic longitude network in our country. The longitude differences between the points of the basic network would be determined with respect to the reference point, AOB.

With regard that Belgrade is at approximately the same latitude as Munich, Vienna, Graz, Milano and most of the stations included in ELN the same stars can be observed from it as from these stations. Due to this in the proposal of including Belgrade in ELN the same observational design can be used as that used at a majority of stations of the central ELN part.

Let in the campaign aimed at the determining of the longitude differences between Munich, Belgrade and Graz the same stars following the same programme as in 1988 be observed. Then it should be expected to achieve the same accuracy of a single star

Table 8: Values of reliability measures r_{ii}, G_i^*, $\frac{G_i^*}{\sigma_i}$ and $\sqrt{\lambda_{x,i}}$ longitude differences for Design III.

i	From	To	r_{ii}	G_i^*	G_i^*/σ_i	$\sqrt{\lambda_{x,i}}$
1	1	13	0.3549	0.06463	4.442	3.891
2	13	2	0.5042	0.05775	3.970	3.121
3	2	4	0.3301	0.06034	4.147	3.420
4	4	3	0.3301	0.06034	4.147	3.420
5	3	7	0.7084	0.05027	3.456	2.132
6	7	8	0.5861	0.05374	3.694	2.620
7	8	9	0.5494	0.05406	3.716	2.662
8	9	3	0.5877	0.05288	3.635	2.504
9	3	5	0.6369	0.05612	3.857	2.922
10	5	6	0.3788	0.06850	4.708	4.295
11	6	10	0.3565	0.06850	4.708	4.295
12	10	11	0.4940	0.05099	4.193	3.494
13	11	12	0.4936	0.06080	4.180	3.472
14	12	2	0.5277	0.05931	4.077	3.303
15	2	1	0.6370	0.05212	3.583	2.401
16	1	3	0.4711	0.05764	3.962	3.107
17	3	14	0.3275	0.06057	4.163	3.446
18	14	7	0.3275	0.06057	4.163	3.446
19	7	9	0.4807	0.06015	4.134	3.398
20	9	5	0.4787	0.06024	4.141	3.409
21	5	11	0.5185	0.05772	3.968	3.116
22	11	8	0.6092	0.05449	3.745	2.717
23	8	10	0.4895	0.06001	4.125	3.383
24	10	12	0.4111	0.06376	4.383	3.798
25	12	13	0.4110	0.07755	5.331	4.091

transit and the same accuracy in the determination of longitudes and longitude differences as in the campaign Munich-Vienna-Graz.

Based on this and observational geometry for the longitude network the precision and reliability of the network for including AOB in ELN can be examined.

Four functional models are considered (observational designs) for the purpose of estimating the reliability of including Belgrade in ELN. They differ according to the linear-model design, whereas the stochastic model for describing individual observations (in this case estimation of longitude differences) is the same.

4.1. FUNCTIONAL MODELS

The network comprises 14 stations, 13 stations included in ELN and Belgrade, with 22 longitude differences (Design I), 24 (Design II), 25 (Design III) and 28 (Design IV).

Table 9: Values of reliability measures r_{ii}, G_i^*, $\frac{G_i^*}{\sigma_i}$ and $\sqrt{\lambda_{x,i}}$ longitude differences for Design IV.

i	From	To	r_{ii}	G_i^*	G_i^*/σ_i	$\sqrt{\lambda_{x,i}}$
1	1	13	0.3805	0.06384	4.388	3.806
2	13	2	0.5515	0.05639	3.876	2.955
3	2	4	0.5105	0.05557	3.820	2.855
4	4	3	0.5925	0.05293	3.638	2.512
5	3	7	0.7137	0.05013	3.446	2.111
6	7	8	0.6073	0.05309	3.649	2.533
7	8	9	0.5543	0.05380	3.698	2.628
8	9	3	0.5908	0.05269	3.622	2.479
9	3	5	0.6689	0.05253	3.611	2.457
10	5	6	0.5146	0.05773	3.968	3.118
11	6	10	0.4527	0.06102	4.194	3.496
12	10	11	0.4998	0.05896	4.053	3.262
13	11	12	0.5305	0.05892	4.050	3.257
14	12	2	0.5560	0.05789	3.980	3.137
15	2	1	0.6370	0.05181	3.561	2.357
16	1	3	0.5044	0.05602	3.851	2.911
17	3	14	0.6844	0.05111	3.513	2.257
18	14	7	0.5149	0.05642	3.878	2.959
19	7	9	0.4961	0.05900	4.055	3.266
20	9	5	0.4872	0.05937	4.081	3.310
21	5	11	0.5254	0.05744	3.948	3.083
22	11	8	0.6143	0.05430	3.733	2.694
23	8	10	0.5055	0.05832	4.009	3.187
24	10	12	0.4661	0.05997	4.122	3.378
25	12	13	0.4573	0.06100	4.193	3.494
26	13	14	0.4389	0.06186	4.252	3.590
27	14	4	0.4776	0.06015	4.134	3.398
28	4	6	0.4664	0.06065	4.169	3.455

For each linear model one calculates the local-reliability coefficients of observations r_{ii}, the marginal gross errors G_i^*, the normed marginal gross errors G_i^*/σ_i and distorsion parameters $\sqrt{\lambda_{x,i}}$. The obtained values are given in Tables 6, 7, 8 and 9.

For all linear models: Design I, Design II, Design III and Design IV the local-reliability coefficients are higher than 0.2, a value assumed as a lower limit in the case of such networks (optimal value 0.4). The normed marginal gross errors are also less than the limit of 7.65 indicating that all these designs satisfy reliability criteria.

The reliability measures show that among the four examined linear models the maximal homogeneity of the observed longitude differences is achieved with linear model IV. In the case of this model the individual determinations of longitude differences are not included and every network station is included with at least three longitude differences.

Since all the stars of the observing programme corresponding to the 1988 campaign can be observed from Belgrade, for the stochastic model it is possible to take the data from the campaign of determining the longitude differences Munich-Vienna-Graz. The reliability of including Belgrade in ELN depends then on the geometry of the linear model.

5. CONCLUSION

The best (adequate) mathematical adjustment model, functional - FM3 and stochastic (weight model) - P_{KD3}, is obtained so that the systematic and random influences in the measurements and star positions on the determining of longitudes and longitude differences are reduced to a negligible value compared to the standard errors of these quantities.

There is a systematic difference between the two reference frames given by the catalogues FK5 and HIPPARCOS.

The difference of the reference frames affects the longitude determination, but not also that of the longitude differences.

The including of Belgrade in ELN can be achieved with a high reliability and satisfactory precision through a campaign of the longitude-difference determining with two nearby stations, ELN members.

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